



Lazar, I., & Radu, A. (2018). *Reliability of vibration-supressed structures under seismic loads*. Paper presented at Eleventh U.S. National Conference on Earthquake Engineering, Los Angeles, United States.

Peer reviewed version

[Link to publication record in Explore Bristol Research](#)
PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>



Eleventh U.S. National Conference on Earthquake Engineering
Integrating Science, Engineering & Policy
June 25-29, 2018
Los Angeles, California

RELIABILITY OF VIBRATION-SUPPRESSED STRUCTURES UNDER SEISMIC LOADS

I. F. Lazar¹ and A. Radu²

ABSTRACT

This paper investigates the reliability of multi-storey buildings subjected to seismic loads and equipped with tuned-inerter-damper (TID) vibration suppression systems. The inerter, equivalent to a capacitor in the electrical domain, produces a force proportional to the relative acceleration across its two terminals. The TID is a newly developed vibration suppression system, where the inerter replaces the mass element of a traditional tuned-mass-damper (TMD). The studies carried out to date focused on the use of TIDs modelled inside single or multi-storey linear host structures, generally subject to simplified loading patterns, such as sinusoidal waves or singular earthquake time-history records.

Although previous work aided the understanding of the TID dynamics and showed its advantages over alternative passive control systems (such as TMDs and viscous dampers), it is necessary to perform more in-depth studies, where the simplifying assumptions are gradually dropped. Numerical examples of structures modelled as non-linear multi-degree-of-freedom (MDOF) systems, equipped with TIDs, and subjected to synthetic ground-motion records, are shown. The effects of using the proposed vibration suppression system are presented in terms of reliability metrics, such as crossing rates of displacements over critical thresholds, probability of failure and time duration until the failure. In addition, the differences in the structural responses of the controlled and uncontrolled MDOF systems is shown for the NGA-West dataset of earthquake records.

¹Senior Research and Teaching Associate, Department of Mechanical Engineering, University of Bristol, Bristol, UK, BS8 1TR (email: irina.lazar@bristol.ac.uk)

²Marie-Curie Research Fellow, Department of Civil Engineering, University of Bristol, Bristol, UK, BS8 1TR

Reliability of Vibration-Suppressed Structures Under Seismic Loads

I. F. Lazar¹ and A. Radu²

ABSTRACT

This paper investigates the reliability of multi-storey buildings subjected to seismic loads and equipped with tuned-inerter-damper (TID) vibration suppression systems. The inerter, equivalent to a capacitor in the electrical domain, produces a force proportional to the relative acceleration across its two terminals. The TID is a newly developed vibration suppression system, where the inerter replaces the mass element of a traditional tuned-mass-damper (TMD). The studies carried out to date focused on the use of TIDs modelled inside single or multi-storey linear host structures, generally subject to simplified loading patterns, such as sinusoidal waves or singular earthquake time-history records.

Although previous work aided the understanding of the TID dynamics and showed its advantages over alternative passive control systems (such as TMDs and viscous dampers), it is necessary to perform more in-depth studies, where the simplifying assumptions are gradually dropped. Numerical examples of structures modelled as non-linear multi-degree-of-freedom (MDOF) systems, equipped with TIDs, and subjected to synthetic ground-motion records, are shown. The effects of using the proposed vibration suppression system are presented in terms of reliability metrics, such as crossing rates of displacements over critical thresholds, probability of failure and time duration until the failure. In addition, the differences in the structural responses of the controlled and uncontrolled MDOF systems is shown for the NGA-West dataset of earthquake records.

Introduction

Vibration-control devices are used to increase the reliability of structural systems subjected to seismic loading. The reliability-based design of a passive system, the tuned-inerter damper (TID), is studied in this paper. The inerter was developed by Smith [1] in the 2000s to complete the force-current electrical-mechanical analogy, where an inerter is the equivalent to a capacitor. The ideal inerter was defined as a mechanical two-node (two-terminal), one-port device with the property that the equal and opposite force applied at the nodes is proportional to the relative acceleration between the nodes [1]. Recent studies have considered the use of inerters for civil engineering applications [2,3,4]. In [2,5], the TID was proposed to reduce vibrations in civil engineering structures subjected to base excitation. The numerical study concluded that the performance is comparable to or improves on that of a traditional TMD, with a number of important advantages such as smaller overall device size due to gearing and the capability to damp out high frequency responses away from resonance.

Reliability-based design of dynamic systems subjected to seismic loading is a complex process that is usually resolved by using simplifying assumptions, such as considering that the structural system behaves linearly, or by adopting Gaussian, band-limited white noise as proxies

¹ Senior Research and Teaching Associate, Department of Mechanical Engineering, University of Bristol, Bristol, UK, BS8 1TR (email: irina.lazar@bristol.ac.uk)

² Marie-Curie Research Fellow, Department of Civil Engineering, University of Bristol, Bristol, UK, BS8 1TR

for samples of the ground motion. The only general method for solving such complex problems is using Monte Carlo simulations, which could be computationally prohibitive for complex realistic structural systems subjected to stochastic non-Gaussian, non-stationary processes. The main goal of this paper is to propose an efficient method for the design of TIDs for structural systems under seismic loading. Analytical design of inerter-based dampers has been recently proposed in [6] for linear host structures subjected to stationary Gaussian processes. The method is based on stochastic reduced-order models (SROMs) [7], i.e. stochastic processes that have a finite number of samples selected in an optimal manner from the samples of the target process.

A practical measure for the reliability of structures is the probability that the maximum absolute responses of the structure exceed critical limits. This measure is used as the design criterion for the parameters of the TID. Since the response of dynamic systems is sensitive to the frequency content of the ground motions, the design of the TID must consider the moment magnitude, m , and the source-to-site distance, r . Simulated ground motions as a function of (m, r) , provided by the specific barrier model described in [8, 9], are used for the design. Numerical results are shown for a nonlinear, three-degree-of-freedom system subjected to simulated ground motions. The system's performance is tested for the ground motion samples available in the PEER NGA-West dataset.

Seismic Hazard Characterization

The seismic hazard at a site can be characterized by earthquakes of various magnitudes produced by different sources around the site. One way to retrieve this information for sites in the United States is by using the USGS Disaggregation tool. Fig. 1 (left panel) shows the contribution of earthquakes characterized by the magnitude m , and the closest distance to the source r , at a site in downtown San Francisco. The earthquakes with parameters (m, r) are characterized by the probability of occurrence $P(m, r)$. For given site conditions (e.g. NEHERP type B-soil, with a shear-wave velocity in the top 30 meters of soil $v_{s30} = 620 \text{ m/s}$), the frequency content of ground motions is controlled by the pair (m, r) . The right panel of Fig. 1 shows the frequency content for several values of (m, r) in terms of the one-sided spectral-density function $g(\nu; m, r)$.

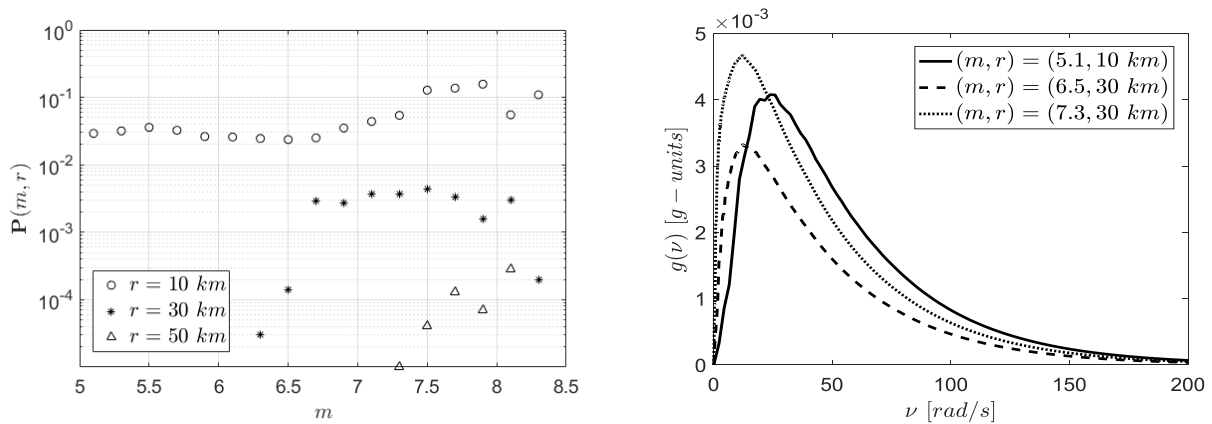


Figure 1. Probabilities $P(m, r)$ of earthquakes with parameters (m, r) (left); one-sided spectral-density function spectral densities $g(\nu; m, r)$ (right).

Functions $g(\nu; m, r)$ are the output of the specific-barrier model (SBM) [8, 9], which is a global

seismological model that characterizes the frequency content of the ground motions as a function of (m, r) , local soil conditions v_{s30} , and seismological regime. Furthermore, ground-motion time histories $\{y_k(t), k = 1, \dots, n\}$ are simulated for each (m, r) , as samples of a zero-mean non-Gaussian, non-stationary stochastic process $Y(t)$ defined by

$$Y(t) = f(t)Y_s(t), 0 \leq t \leq t_f, \quad (1)$$

where t_f is the duration of the ground motion, $f(t)$ is a deterministic time-modulation function

$$f(t) = \alpha t^\beta e^{-\gamma t}, \quad (2)$$

where t_f and the scalar parameters α, β, γ are also provided by the SBM; and $Y_s(t)$ is a zero-mean, stationary, non-Gaussian process with second-order moment properties given by $g(v; m, r)$. For the current study, a Student's T marginal distribution with parameters fitted to accommodate a kurtosis value of 14.3 for ground motions, characteristic for the NEHERP type B-soil ground motions in the NGA West dataset, is considered.

Structural System

Uncontrolled Structure

Let $X(t) = \{X_1(t), X_2(t), X_3(t)\}$ be the relative displacement vector of a three-degree-of-freedom (3DOF) system subjected to the ground acceleration $Y(t)$. The vector process $X(t)$ satisfies equations

$$M\ddot{X}(t) + C\dot{X}(t) + K(X(t) + \varepsilon X(t)^3) = -M\underline{1}Y(t) \quad (3)$$

for a Duffing dynamic system, where M, C and K are square matrices of dimension N for the mass, damping and stiffness of the system, $\underline{1}$ is a $(N, 1)$ unit vector, and ε is a parameter controlling the amount of nonlinearity in the restoring force. Figure 2 shows the backbone curves for the $N=3$ DOF uncontrolled Duffing system. These were obtained by varying amplitude of the harmonic forcing of the type $A \sin \omega t$ from $A = 0.05g$ to $A = 1.2g$, where g represents the gravitational acceleration. This span is relevant in civil engineering applications, as it covers the entire ground motion intensity range, from not felt (with no associated damage) to extreme (with very heavy potential damage). It can be seen that, as the forcing amplitude increases, the peak response shifts to the right (hardening) and the system's nonlinearity becomes more severe.

Structure equipped with TIDs

Previous research has shown that TIDs are most effective when installed at bottom-storey level [2], hence we will consider that the host structure is equipped with a single TID system, mounted between the ground and the first storey. In this case, the vector process $X(t)$ must satisfy equations

$$M_d\ddot{X}(t) + C_d\dot{X}(t) + K_dX(t) + \varepsilon\underline{1}_{NL}X(t)^3 = -M_d\underline{1}_dY(t) \quad (4)$$

where M_d, C_d and K_d are the mass, damping and stiffness matrices of the TID-controlled system,

$\underline{1}_{NL}$ is a square matrix of dimension $(N + 1)$ with the first N diagonal terms equal to unity and all other terms equal to zero, and $\underline{1}_d$ is a $(N + 1, 1)$ unit vector. Note that the controlled system has one extra DOF, introduced by the TID. The controlled system's matrices are

$$M_d = \begin{bmatrix} M & O_{N,1} \\ O_{1,N} & \mu M_{1,1} \end{bmatrix}; C_d = \begin{bmatrix} C_{1,1} + c_d & C_{1,2} & 0 & -c_d \\ C_{2,1} & C_{2,2} & C_{2,3} & 0 \\ 0 & C_{3,2} & C_{3,3} & 0 \\ -c_d & 0 & 0 & c_d \end{bmatrix};$$

$$K_d = \begin{bmatrix} K_{1,1} + k_d & K_{1,2} & 0 & -k_d \\ K_{2,1} & K_{2,2} & K_{2,3} & 0 \\ 0 & K_{3,2} & K_{3,3} & 0 \\ -k_d & 0 & 0 & k_d \end{bmatrix}. \quad (5)$$

where $M_{i,j}$, $C_{i,j}$ and $K_{i,j}$ are the (i, j) terms of the mass, damping and stiffness matrices of the uncontrolled system, $c_d = 2\zeta_d \nu_d \mu M_{1,1}$ is the TID damping, $k_d = \nu_d^2 \mu M_{1,1}$ is the TID stiffness, ζ_d is the TID damping ratio, ν_d is the TID fundamental frequency and μ is the inertance-to-mass ratio between the TID and the host structure.

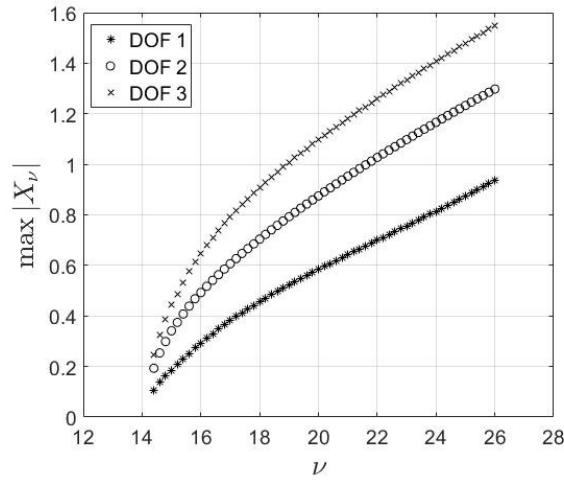


Figure 2. Backbone curves for a 3 DOF duffing oscillator subjected to ground acceleration

For the current study, the inertance-to-mass ratio is assumed to be fixed to a value of $\mu = 0.25$, since this parameter is usually chosen by the designer of the system on economic considerations. The response of the TID-controlled system is expected to decrease as μ increases [2]. Thus, the design of the TID is focused on finding some optimal values for (ζ_d, ν_d) , for the given value of μ . This optimization, aimed at minimizing the relative displacement of the host structure, is done numerically.

Reliability-Based Design of the TID

The main goal of the paper is to introduce a general, efficient framework to design the TID's parameters (ζ_d, ν_d) based on the reliability of the performance of the controlled structure. The only reliable method that can fulfil this job for non-linear systems subjected to non-stationary, non-Gaussian input is Monte Carlo (MC), which involves a large number of deterministic dynamic

analyses of the controlled system, and is computationally prohibitive for realistic, complex systems. The method proposed here is based on stochastic reduced-order models (SROM) for the ground-motions input, which are describes in the next section.

Stochastic Reduced Order Models

The stochastic reduced order model (SROM) [7] can be viewed as a “smart” Monte Carlo method. Like MC simulation, the method uses samples of the ground acceleration process to characterize the structural response. In contrast to MC, which uses a large number n of samples selected at random, it uses a small number \tilde{n} of samples selected in an optimal manner. A stochastic reduced order model $\tilde{Y}(t)$ of $Y(t)$ is a stochastic process with $\tilde{n} \ll n$ samples $\{\tilde{y}_k(t), k = 1, \dots, \tilde{n}\}$ extracted from samples of $Y(t)$. Usually, the samples of $\tilde{Y}(t)$ are not equally likely. Pairs $\{(\tilde{y}_k(t), p_k), k = 1, \dots, \tilde{n}\}$, where p_k are the probabilities of samples $\tilde{y}_k(t)$, define completely the probability law of $\tilde{Y}(t)$.

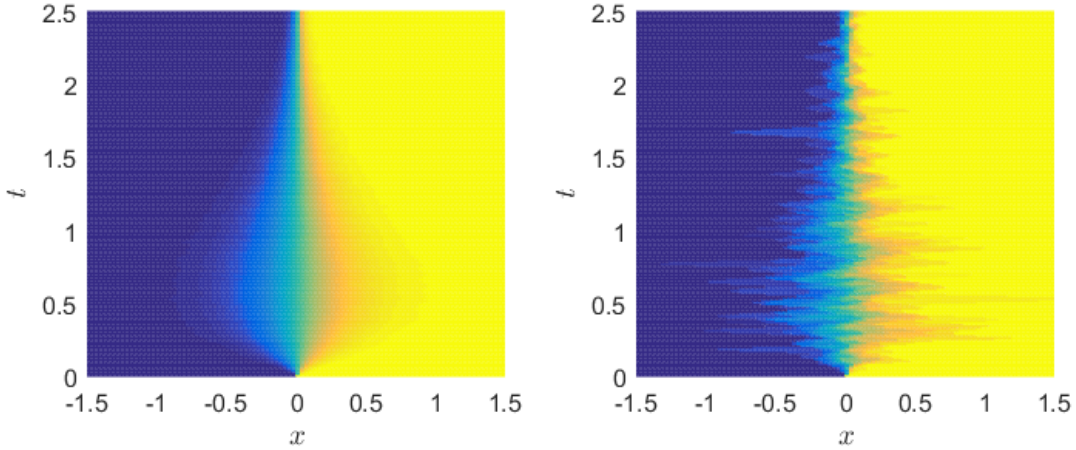


Figure 3. Marginal distributions of $Y(t)$ (left) and its SROM $\tilde{Y}(t)$ with for $\tilde{n} = 20$ (right), for $(m, r) = (5.1, 10km)$.

To construct $\tilde{Y}(t)$, samples of $Y(t)$ are selected such that the discrepancies between the probability laws – in this paper, quantified in terms of their marginal distributions, high-order moments and covariance functions – are minimized. The marginal distributions, q -order moments and the covariance function of $\tilde{Y}(t)$ are:

$$\tilde{F}(y; t) = \mathbb{P}[\tilde{Y}(t) \leq y] = \sum_{k=1}^{\tilde{n}} p_k \mathbb{1}\{\tilde{y}_k(t) \leq y\}, \quad (6)$$

$$\tilde{\mu}(q; t) = E[\tilde{Y}(t)^q] = \sum_{k=1}^{\tilde{n}} p_k \tilde{y}_k(t)^q, \quad (7)$$

$$\tilde{\rho}(t, s) = E[\tilde{Y}(t), \tilde{Y}(s)] = \sum_{k=1}^{\tilde{n}} p_k \tilde{y}_k(t) \tilde{y}_k(s). \quad (8)$$

Figs. 3, 4 and 5 show the marginal distributions, first- and second-order moments, and the covariance function of the process $Y(t)$, calculated using MC with $n = 10,000$ samples, versus the SROM estimates of $\tilde{Y}(t)$, calculated for $\tilde{n} = 20$, for parameters $(m, r) = (5.1, 10km)$. The statistics of $\tilde{Y}(t)$ cannot match perfectly the statistics of the original process $Y(t)$, but they provide a reasonable proxy for it, given the small number of samples it uses. The mapping from the samples

$\tilde{y}_k(t)$, $k = 1, \dots, \tilde{n}$ of the SROM $\tilde{Y}(t)$ to the response samples $\tilde{x}_k(t)$ is done by solving Eqs. (3) and (4) for each sample of the SROM. The response samples $\tilde{x}_k(t)$, $k = 1, \dots, \tilde{n}$ are also weighed by the probabilities p_k . The accuracy of the structural performance evaluated through the samples $\tilde{x}_k(t)$ is guaranteed by the construction of the SROM $\tilde{Y}(t)$. The accuracy of the SROM $\tilde{Y}(t)$ depends on the number of samples \tilde{n} , and on the optimization process of minimizing the differences between the probability laws of the original and the SROM processes.

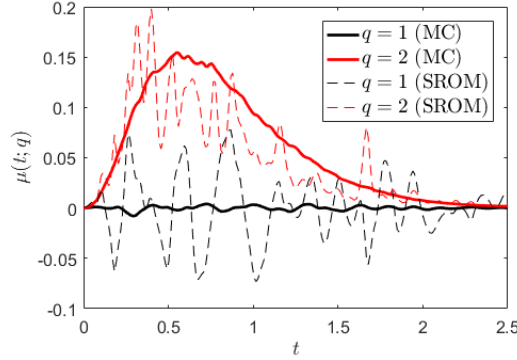


Figure 4. First-order (black) and second-order (red) moments of $Y(t)$ and its SROM $\tilde{Y}(t)$ with $\tilde{n} = 20$ for $(m, r) = (5.1, 10km)$.

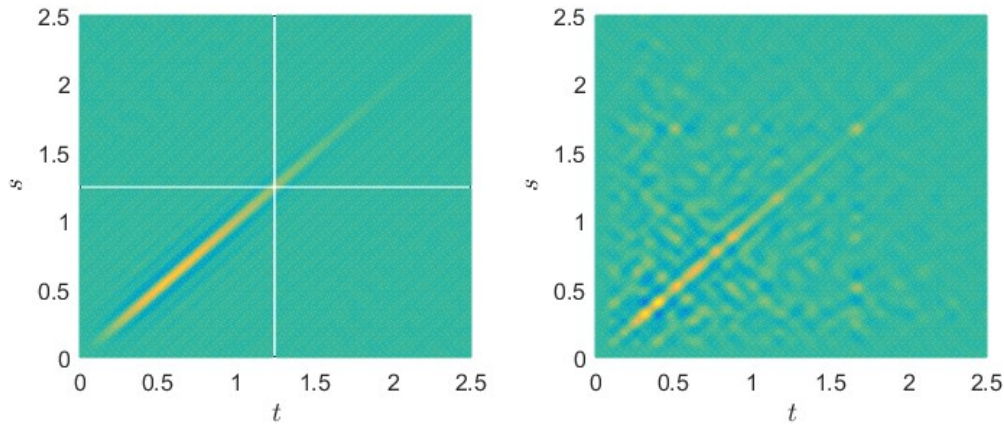


Figure 5. Covariances of $Y(t)$ (left) and its SROM $\tilde{Y}(t)$ with for $\tilde{n} = 20$ (right), for $(m, r) = (5.1, 10km)$.

The tail distributions of the response process, that is, the probability that the maximum absolute response $\max_{t \geq 0} |X(t)|$ exceeds a given value x , is calculated using the samples $\tilde{x}_k(t)$ as follows:

$$\mathbf{P}\left(\max_{t \geq 0} |X(t)| > x\right) = \sum_{k=1}^{\tilde{n}} p_k \mathbf{1}\left(\max_{t \geq 0} |\tilde{x}_k(t)| > x\right), \quad (9)$$

where $\mathbf{1}(\cdot)$ is the indicator function. The tail-distribution functions for the uncontrolled and the TID-controlled response of the systems in Eqs. (3) and (4) are calculated using both the MC and the SROM methodologies. Results are shown in the left panel of Fig. 6, for $(m, r) = (6.5, 30km)$,

and a set of parameters (ζ_d, ν_d) for the TID (red lines) and for the uncontrolled system (black lines). It is seen that the SROM provides a good approximation of the exceedance probability to the MC solution, with just a fraction of the computational effort. A reduction in the response of the controlled system observed in the results provided by both methods. The right panel of Fig. 6 shows the SROM tail distributions of the maximum absolute response of the TID-controlled system for $(m, r) = (6.5, 30km)$ and the entire range of parameters analyzed, that is, $\zeta_d \in [0, 0.3]$ and $\nu_d \in [0, 50] \text{rad/s}$.

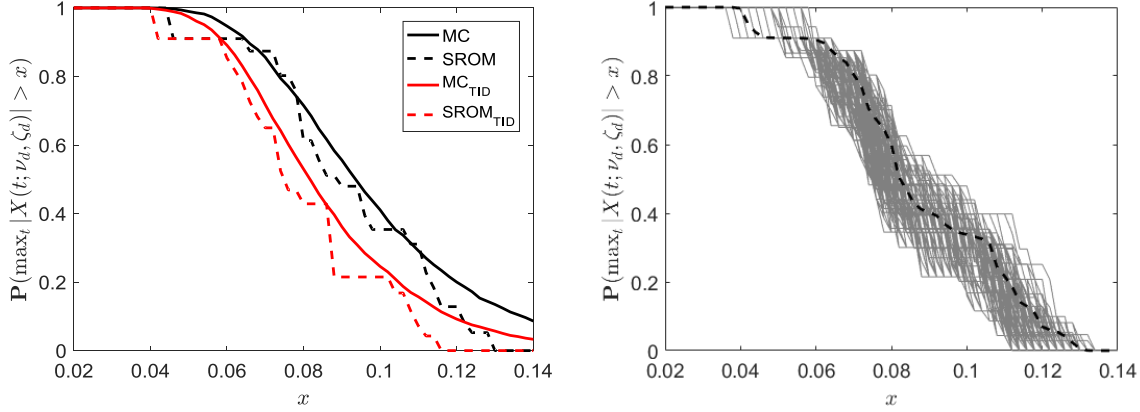


Figure 6. Tail distributions of the maximum absolute response for the uncontrolled (red lines) and TID-controlled (black lines) systems using MC and SROM, respectively for the ground motion characterized by $(m, r) = (6.5, 30km)$ (left); tail distributions of the maximum absolute response of the TID-controlled system using SROM for the ground motion characterized by $(m, r) = (6.5, 30km)$ and the full range of parameters $\zeta_d \in [0, 0.3]$ and $\nu_d \in [0, 50] \text{rad/s}$.

The selection of parameters (ζ_d, ν_d) for the design of the TID for a given value (m, r) is done by following four steps:

Step 1: Construct a SROM $\tilde{Y}(t)$ of $Y(t)$ for the given parameters (m, r) ;

Step 2: Calculate the response $\tilde{X}(t; \zeta_d, \nu_d)$ of the TID-controlled system for each pair of parameters (ζ_d, ν_d) ;

Step 3: Calculate the $P\left(\max_{t \geq 0} |X(t; \zeta_d, \nu_d)| > x\right)$ using $\tilde{X}(t; \zeta_d, \nu_d)$ for each pair of parameters (ζ_d, ν_d) ;

Step 4: Select the values (ζ_{d0}, ν_{d0}) for which the area $\int_x P\left(\max_{t \geq 0} |X(t; \zeta_d, \nu_d)| > x\right) dx$ is minimum. Note that the value $\int_x P\left(\max_{t \geq 0} |X(t; \zeta_d, \nu_d)| > x\right) dx$ is a measure of the overall probability of exceedance of the response for a range of critical values x .

Fig. 7 shows the values for $\int_x P\left(\max_{t \geq 0} |X(t; \zeta_d, \nu_d)| > x\right) dx$ for all TID design parameters pairs (ζ_d, ν_d) , corresponding to the selected inertance-to-mass ratio $\mu = 0.25$, and the ground motions characterized by $(m, r) = (6.5, 30km)$. The minimum is obtained for the pair $(\zeta_{d0} = 0.085, \nu_{d0} = 14.45 \text{rad/s})$. It must be considered that the design is performed by using an approximate method, and therefore a value in a small vicinity of (ζ_{d0}, ν_{d0}) is acceptable for design purposes.

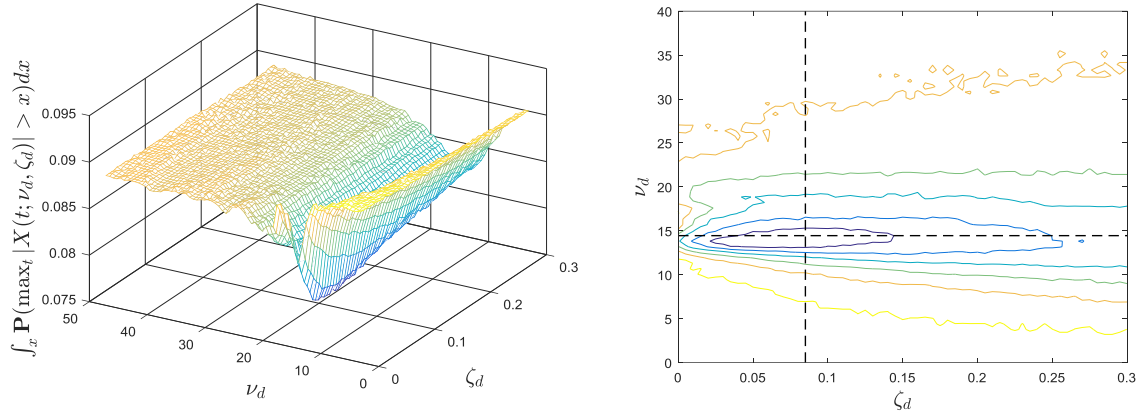


Figure 7. Areas $\int_x \mathbf{P}\left(\max_{t \geq 0} |X(t; \zeta_d, \nu_d)| > x\right) dx$ for the full range of the TID parameters (ζ_d, ν_d) , and the ground motion characterized by $(m, r) = (6.5, 30\text{km})$.

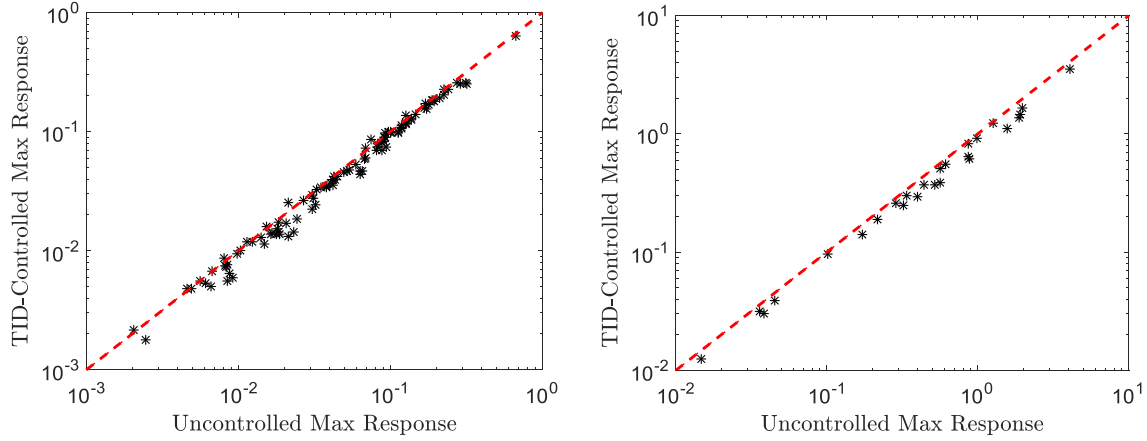


Figure 8. Response of the maximum absolute response of the uncontrolled vs. TID-controlled systems with (ζ_{d0}, ν_{d0}) designed for $(m, r) = (5.1, 10\text{km})$ (left) and $(m, r) = (6.5, 30\text{km})$ (right), subjected to corresponding motions from the NGA-West dataset.

The left and right panels of Fig. 8 show plots of the maximum absolute responses of the uncontrolled vs. the TID-controlled systems in Eqs. (3) and (4) with (ζ_{d0}, ν_{d0}) designed for $(m, r) = (5.1, 10\text{km})$ and $(m, r) = (6.5, 30\text{km})$, subjected to the real ground-motion time histories from the NGA-West dataset, corresponding to the range of $(m \in [5.0, 5.2], r \in [0, 20]\text{km})$ and $(m \in [6.4, 6.6], r \in [20, 40]\text{km})$, respectively. The response of the TID-controlled system was improved in approximately 90% of the motions selected for the case of $(m, r) = (5.1, 10\text{km})$, and for all motions selected for the case $(m, r) = (6.5, 30\text{km})$. Note that the number of records in the range $(m \in [6.4, 6.6], r \in [20, 40]\text{km})$ is substantially smaller than for the range $(m \in [5.0, 5.2], r \in [0, 20]\text{km})$.

Conclusions

A novel, highly efficient methodology based on stochastic reduced-order models (SROM) was proposed for the design of the tuned-inerter dampers (TID) modelled inside a three-degree-of-freedom nonlinear structure, subjected to seismic excitation, using reliability metrics of the system response. Like Monte Carlo, the method is general and can be applied to any type of nonlinear system subjected to any random input. A step-by-step algorithm for the design of the TID parameters is presented and numerical examples are shown for simulated ground motions. The TID-controlled system designed for the random vibration is tested for real ground-motion records, for which the response is reduced for the majority of cases.

Acknowledgments

The work reported in this paper has been partly supported by the Marie Skłodowska-Curie Actions of the European Union's Horizon 2020 Program under the grant agreement 704679 - PARTNER. This support is gratefully acknowledged.

References

1. Smith MC. Synthesis of mechanical networks the inerter. *IEEE Transactions on Automatic Control* 2002; **47**: 1648-1662.
2. Lazar IF, Neild SA, Wagg DJ. Using an inerter-based device for structural vibration suppression. *Earthquake Engineering and Structural Dynamics* 2014; **43**(8): 1129-1147.
3. Marian L, Giaralis A. Optimal design of a novel tuned mass-damper-inerter (TMDI) passive vibration control configuration for stochastically support-excited structural systems. *Probabilistic Engineering Mechanics* 2014; **38**: 156 -164.
4. Takewaki I, Murakami S, Yoshitomi S, Masaaki T. Fundamental mechanism of earthquake response reduction in building structures with inertia dampers. *Structural Control and Health Monitoring* 2012; **19**.
5. Lazar IF, Neild SA, Wagg DJ. Vibration suppression of cables using tuned inerter dampers; *Engineering Structures* 2016; **122**: 62-71.
6. Giaralis A, Taflanidis A. Optimal tuned mass-damper-inerter (TMDI) designed for seismically excited MDOF structures with model uncertainties based on reliability criteria; *Struct Control Health Monit.* 2017; DOI: 10.1002/stc.2082.
7. Grigoriu M. Reduced order models for random functions. application to stochastic problems. *Applied Mathematical Modelling* 2009; **33**: 284-293.
8. Halldorsson B, Papageorgiou AS. Calibration of the specific barrier model to earthquake to different tectonic regions. *Bulletin of Seismological Society of America* 2005; **95**(4): 1276–1300.
9. Radu A and Grigoriu M. A Site-Specific Seismological Model for Probabilistic Seismic-Hazard Assessment. *Bulletin of Seismological Society of America* 2014; **104**(6): 3054–3073.